Comparison of experimental data for maximal lateral pressures obtained for beds of various relative height with results obtained by computation using Eq. (2) indicated insignificant discrepancies (Fig. 3). Therefore, it is possible to recommend the application of the above expression for calculations on stresses in aerated loose material beds during their forced motion along vertical pipes.

NOTATION

D, diameter of the pipeline; f, resistance coefficient; h, loose material bed height above the piston; n, lateral pressure coefficient; σ, vertical pressure; σ_b, loose material pressure on pipeline walls; ω, gas flow filtration velocity through the loose material bed; ω_{cr}, gas flow filtration velocity at which the fluidization effect for a given material begins; dP/dh, pressure gradient due to gas flow filtration through the loose material bed; γ_v volumetric weight of loose material.

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PRESSURE LOSS DURING FLUID FLOW IN A CHANNEL ROTATING PERPENDICULARLY TO THE AXIS OF ROTATION

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Results are presented which were obtained in experiments to determine the hydraulic resistance of a channel during its rotation around an axis perpendicular to the channel axis.

In the design of cooling systems for rotating objects, one needs data on the hydraulic resistance of channels whose axes are perpendicular to the axis of rotation. Studies that have been made [1, 2, 3] gave good agreement of results only for low rates of rotation where Coriolis forces have a controlling effect on flow [4].

At high rates of rotation, i.e., in the region where centrifugal forces have a controlling influence, considerable disagreement is observed in the data from available experimental studies [2, 5, 6]. Because of this, the performance of further experiments is advisable.

This paper presents the results of a study of pressure loss in a straight, technically smooth channel of circular cross section which is arranged perpendicularly to the axis of rotation, being a section of a rectangularly shaped rotating system, and which is included in a circulation loop. To perform the experiments, a special device was constructed with rotational speeds up to 1000 rpm having a 130-mm mean radius of channel rotation and a hydraulic system which provided the required flow rates and pressures of the working medium (water and transformer oil were used in the experiments).

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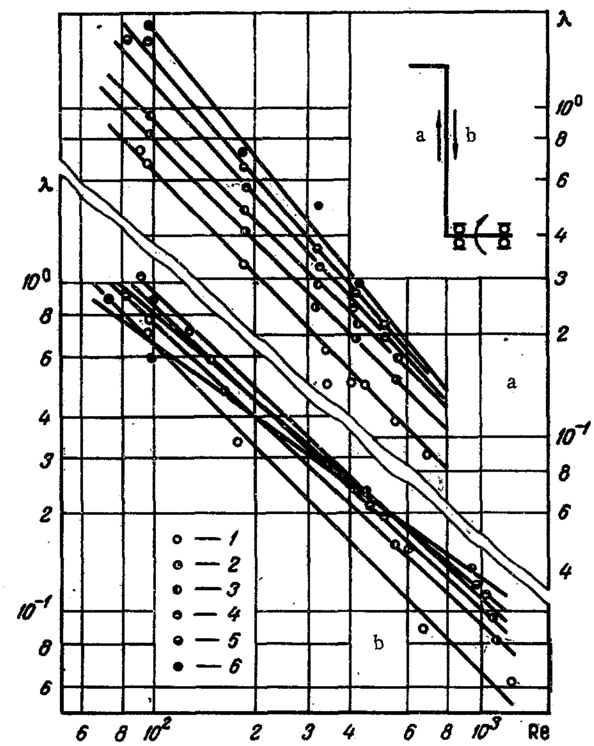

Fig. 1. Dependence of the coefficient of friction λ on Reynolds number Re and rate of rotation for Re = 100-1000 [transformer oil: a) centrifugal flow; b) centripetal flow]: 1) n = 0 rpm; 2) 200 rpm; 3) 400 rpm; 4) 600 rpm; 5) 800 rpm; 6) 1000 rpm.

Pressure losses were measured in a section of the channel sufficiently removed from the entrance (the length of the nonworking section was 37 channel diameters for both centrifugal and centripetal flows). The magnitude of loss was established from the difference in static pressures in measurement sections. Four-point loops for pressure takeoff, which were installed in the measurement sections, were connected by tubing to special rotating pressure transducers of the cuff type, the chambers of which were connected to piezometers. The flow rate of the working medium through the channel was determined by the volume method. Temperature measurement at the entrance and exit of the device (to monitor isothermicity of flow) was accomplished by means of thermocouples. The rate of rotation was measured with an ST-5 strobotachometer.

Using the experimental data, a relationship was constructed for the dependence of the coefficient of friction λ on the Reynolds number Re for flow rate at a fixed value of the rate of rotation (the Reynolds number for circumferential velocity $Re_\omega = \omega d^2/2\nu$). Experiments were performed at Reynolds numbers Re = 100-20,000 and rates of rotation up to 1000 rpm ($Re_{\omega\,max} = 840$).

The accuracy of the instruments used made it possible to make the following evaluations of the maximum relative error of derived quantities: $\delta Re \approx 3\%$, $\delta Re_\omega \approx 4.5\%$, $\delta\lambda \approx 12\%$ for minimum flow rates of the working medium and $\delta Re \approx 4.0\%$, $\delta Re_\omega \approx 4.5\%$, $\delta\lambda \approx 4\%$ for maximum flow rates. Variation in the rate of channel rotation had no effect on the accuracy of flow-rate and pressure-drop measurements.

The experimental results are presented in Fig. 1 (oil, Re = 100-1000) and Fig. 2 (water, Re = 600-20,000) for centrifugal and centripetal flows.

In analyzing the data obtained, it is necessary to keep in mind the fact that the effect of rotation on flow is different in the regions where Coriolis or centrifugal forces have a controlling influence [4] with the amount of influence of these forces on flow depending on the rate of rotation and also on the initial mode (in the absence of rotation) of fluid motion.

In the region Re < 2300, the initial flow has a parabolic profile and the distribution of Coriolis forces over a channel cross section is extremely nonuniform. Thus Coriolis forces have a controlling influence on flow at low rates of rotation in this case.

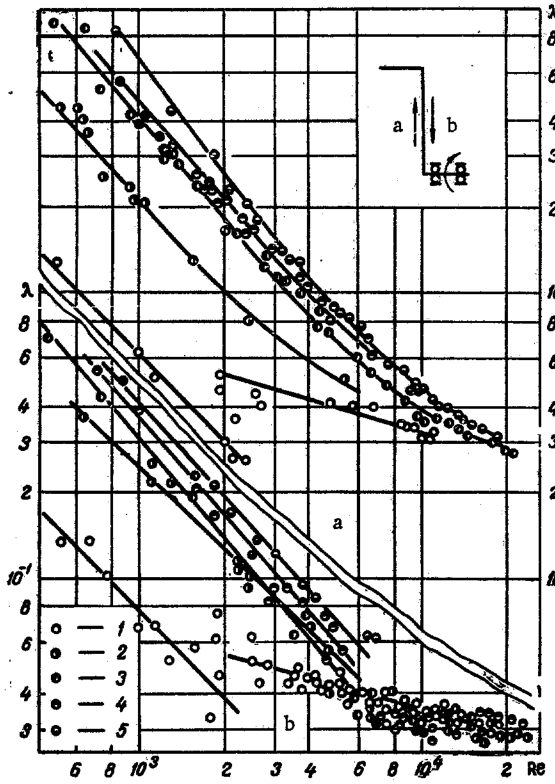

Fig. 2. Dependence of the coefficient of friction λ on Reynolds number Re and rate of rotation for Re = 600–20,000 [water; a) centrifugal flow; b) centripetal flow]. See Fig. 1 for notation.

Because of the nonuniformity in the cross-sectional plane, a vortex pair is formed which leads to a change in the flow-rate profile; the profile becomes fuller (flatter) and the velocity gradient near the wall increases. This gives an increase in resistance so that such laminar flow with macrovortices has a higher coefficient of friction than purely laminar flow. Since the intensity of secondary vortices and the increase in resistance produced by them depend on the rate of rotation, the experimental data for a rotating channel are described by the relation $\lambda = A/Re^m$ (A and m are functions of the number of revolutions). The increase in resistance in the region where Coriolis forces have a controlling influence is the same for both centrifugal and centripetal flows and is more strongly expressed in the region of higher Reynolds numbers for fixed rate of rotation.

To distinguish the regions where Coriolis or centrifugal forces have a controlling influence [4] when Re < 2300, we use the ratio between the centrifugal force at the mean radius of rotation R_{av} of a measurement section and the Coriolis force on the channel axis, $\bar{F} = 0.25\omega R_{av} d/\nu Re$.

The experimental points corresponding to the region where Coriolis forces have a controlling influence ($\bar{F} < 1$) satisfactorily fit (Fig. 3) the expression

$$\bar{\lambda} = 0.13 N_C^{0.45} + 0.25, \quad (1)$$

proposed by V. Shchukin ($\bar{\lambda}$ is the ratio between the coefficients of friction for rotating and stationary channels; $N_C = Re\sqrt{\omega d/w}$ is a criterion which takes into account the effect of Coriolis forces on flow). This equation was based on the data of Trefethen [1] and describes the linear envelope $\bar{\lambda} = f(N_C)$ corresponding to fixed values of Re_ω [3]. Our data, like the results of the experiments in [3], also include points which depart from the asymptote (1) and yield a relation of the form $\bar{\lambda} = f(Re_\omega)$.

Centrifugal forces have a controlling influence on flow for high rates of rotation and initial laminar flow ($\bar{F} > 1$). These same forces are controlling when Re > 2300, since the uniform profile of axial velocities weakens the intensity of the vortex pair under these conditions [4].

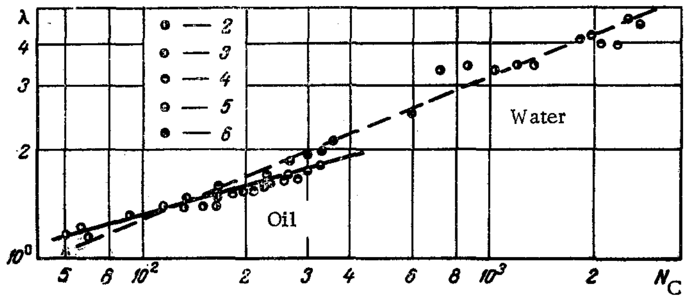

Fig. 3. Dependence of relative coefficient of friction $\bar{\lambda}$ on the quantity $N_C = \mathrm{Re}\sqrt{\omega d/w}$ for the region where Coriolis forces have a controlling influence. See Fig. 1 for notation.

As follows from Figs. 1 and 2, the experimental data for fluid motion toward and away from the center of rotation are somewhat different in the region where centrifugal forces have a controlling influence. This difference is evidently explained by the fact that centrifugal forces, being directed along the flow and in opposition to it, affect the flow region near the wall differently. When the direction of centrifugal forces and the direction of flow are the same, one can expect an increase in flow velocity in the wall region; this leads to an increase in velocity gradient and frictional resistance. With centripetal flow, centrifugal forces facilitate deceleration of fluid particles near the wall, and one can observe either a decrease in velocity gradient (reduction of resistance in comparison with the region where Coriolis forces have a controlling influence) or separation of flow from the wall and the appearance of macrovortices (further increase in resistance) depending upon particle flux and flow velocity in the wall region.

When $\mathrm{Re} > 2300$, centrifugal forces affect not only average flow, but also velocity pulsations (suppression of turbulent pulsations by centrifugal forces was observed visually [7] and was confirmed [8] by direct thermoanemometric measurements). The fact that frictional resistance also increases when $\mathrm{Re} > 2300$ is evidence of the greater role of centrifugal forces in the wall region (change of profile of average velocity) as compared to the direct suppression of pulsations by these forces.

The effect of centrifugal forces on pulsation and average velocity profile leads to an increase in flow stability (the critical value of the Reynolds number increases) and to a change in the nature of the transition (the transition becomes more ordered).

When the fluid moves away from the center of rotation, a smooth transition is observed from laminar flow and laminar flow with macrovortices to turbulent flow. If one takes as the critical value of the Reynolds number values of Re corresponding to points where the relation $\lambda = f(\mathrm{Re})$ deviates from the form $\lambda = A/\mathrm{Re}^m$, critical Reynolds numbers for centrifugal flow can be determined from

$$\mathrm{Re}_{cr} = 2300\,(1 + 0.0026\,\mathrm{Re}_{\omega}^{0.84}). \tag{2}$$

When $\mathrm{Re} > \mathrm{Re}_{cr}$, pressure losses for centrifugal flow in a rotating channel are greater than in a stationary channel, but this difference decreases as the Reynolds number increases and the experimental data for stationary and rotating channels coincide when $\mathrm{Re} > 290\,\mathrm{Re}_{\omega}^{0.69}$.

In the case of fluid motion toward the center of rotation, the coefficient of friction for a rotating channel obeys a relation of the form $\lambda = A/\mathrm{Re}^m$ up to its intersection with the Blasius line, following which the experimental points lie on that line within the limits of experimental error. Hence, in this case it is advisable to take for Re_{cr} values of the Reynolds number which correspond to the points of intersection of the line $\lambda = A/\mathrm{Re}^m$ with the Blasius line:

$$\mathrm{Re}_{cr} = 2300\,(1 + 0.002\,\mathrm{Re}_{\omega}^{1.12}). \tag{3}$$

When $\mathrm{Re} > \mathrm{Re}_{cr}$, pressure losses for centripetal flow in a rotating channel differ little from the loss in a stationary channel.

For the region where centrifugal forces have a controlling influence and $\mathrm{Re} < \mathrm{Re}_{cr}$, the experimental data obey the relation

$$\bar{\lambda}_{cf} = 0.088\,\mathrm{Re}^{0.46} N_C^{0.33} \tag{4}$$

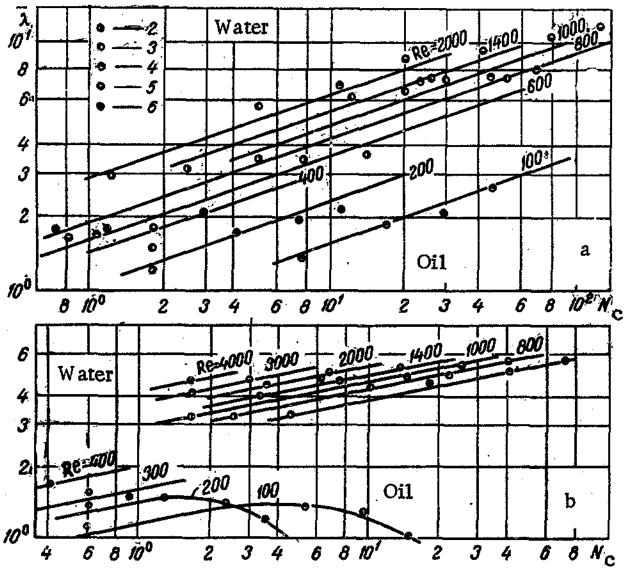

Fig. 4. Dependence of relative coefficient of friction $\bar{\lambda}$ on the quantity $N_c = \omega^2 l d/w^2$ in the region where centrifugal forces have a controlling influence [a) centrifugal flow; b) centripetal flow]. See Fig. 1 for notation.

for centrifugal flow (Fig. 4a) and the relation

$$\bar{\lambda}_{cp} = 0.177\,\mathrm{Re}^{0.4} N_c^{0.185} \tag{5}$$

for centripetal flow (Fig. 4b). The last equation does not describe the function $\bar{\lambda}$ for small Reynolds numbers Re. A comparison of Eqs. (4) and (5) shows that for small values of the criterion $N_c = \omega^2 l d/w^2$ (of the order of 1-10), which characterizes the effect of centrifugal forces on flow, they yield similar values of $\bar{\lambda}$ with $\bar{\lambda}_{cf}$ being somewhat less than $\bar{\lambda}_{cp}$ for $N_c \sim 1$ and somewhat greater for $N_c \sim 10$. For $N_c \sim 100$, $\bar{\lambda}_{cf} > \bar{\lambda}_{cp}$.

For the region $\mathrm{Re} > \mathrm{Re}_{cr}$ and fluid motion away from the center of rotation, the experimental data fit the expression

$$\bar{\lambda}_{cf} = 1 + 1.18 \cdot 10^4\,\mathrm{Re}^{-1.13} N_c^{0.4}. \tag{6}$$

The results obtained in the region where centrifugal forces have a controlling influence occupy an intermediate position with respect to the data in [5, 6].

The dependence recommended in [5] yields results that are highly overestimated in comparison with our results. This is evidently explained by the manner in which the experiment was performed in [5] (the experiment was carried out with an open arrangement for flow of air through a pipe; the resistance of the entrance and exit of the rotating pipe affected the results obtained in this case). This obviously led to the fact that a difference was not observed in [5] between the resistances of a rotating channel for centrifugal and centripetal flows in the region $\mathrm{Re} < \mathrm{Re}_{cr}$.

The qualitative behavior of the relationship $\lambda = f(\mathrm{Re})$, which was obtained for the rotation of a radial channel included in a circulating fluid loop [6], is similar to our data with a differing effect on resistance in the case of centrifugal and centripetal flows also being obtained for the region $\mathrm{Re} < \mathrm{Re}_{cr}$. However, the quantitative growth of resistance for the case presented in [6] is less than follows from Eqs. (4) and (5). The reason for this may be the insufficient length of the stabilization section in the device used in [6] and the location of pressure sampling points only along a single channel generator.

For a turbulent flow mode, i.e., for a relatively weak effect of rotation on resistance, the data obtained by us agree with the results of [2, 5].

Equations (1)-(6) can be used in the design of fluid systems for cooling rotating objects when determining the resistance of straight, technically smooth radial channels of circular cross section for Re = 100-20,000 and Re_ω up to 840 (N_C = 50-3000, N_c = 0.5-100). Furthermore, one should consider what mass forces are dominant in the case under consideration.

Additional experiments should be performed to reveal the effects of roughness, mean radius of rotation, and distance of initial section from axis of rotation on increase in pressure loss.

NOTATION

$Re = wd/\nu$, Reynolds number for flow rate; $Re_{\omega} = \omega d^2/2\nu$, Reynolds number for circumferential velocity; $N_c = \omega^2 l d/w^2$, criterion taking into account the effect of centrifugal forces; $N_C = Re\sqrt{\omega d/w}$, criterion taking into account the effect of Coriolis forces, $\overline{\lambda}$, ratio of the coefficients of friction for rotating and stationary channels; w, flow rate; d, channel diameter; ν, fluid viscosity; ω, angular velocity; l, channel length; R_{av}, mean radius of rotation of channel; λ, coefficient of friction; Re_{cr}, critical value of Reynolds number; $\overline{F}$, ratio of intensities of centrifugal and Coriolis forces; A, m, constants. Indices: cf, cp, centrifugal and centripetal flows.

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A NOTE ON THE THEORY OF THE RANK EFFECT

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The possibility of using the equations of motion of a viscous compressible gas in the Kasterin–Predvoditelev form, to describe the vortex effect, is discussed.

In the theory of vortical flows there exist two basically different approaches. The first is connected with the name of L. Prandtl who assumed that the origination of a vortex takes place in a thin boundary layer, i.e., the nature of origination of the vortex is due to viscosity.

The second approach was advanced by Felix Klein who showed the possibility of origination of a vortex in an ideal liquid as a consequence of discontinuities occurring in the basic hydrodynamic parameters. The idea of Klein was developed by Kasterin [1] who obtained the equations of the vortex field in an ideal liquid, taking into account a discontinuous variation of the hydrodynamic velocity vector. A molecular-kinetic generalization of the equations of Kasterin for a viscous liquid was carried out by Predvoditelev [2, 3, 5].

We recall that to obtain the equations of motion of a viscous liquid in the form of the Navier–Stokes equations, Maxwell had to introduce two hypotheses [4]:

1. In a physically infinitely small volume the transport velocities of two colliding molecules are equal.

2. Between molecules of the gas there act forces of repulsion whose magnitude is inversely proportional to the fifth degree of the distance between the molecules.

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